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Sensitivity analysis of the secondary settling tank double-exponential function model

The secondary settling tank plays a very crucial role in achieving the very strict effluent standards of wastewater treatment plants. To iestigate the ability of the widely used secondary settling tank model, the double-exponential model, to predict the dynamic behavior, a factorial sensitivity analysis was carried out. A typical and a full-scale model were analyzed. Results obtained have indicated that only the parameters v_0, r_p and f_{ns} play a dominant role in determining the suspended solids concentration in the effluent stream (the settler top layer). In contrast, none of the model parameters, or their interactions, is found to affect the suspended solids concentration in the underflow stream (the settler bottom layer). This result clearly indicates that the model has a structural problem regarding the prediction of the suspended solids concentration in the underflow stream.

Keywords: wastewater, activated sludge, secondary settling tank, modeling, sensitivity analysis.

1. Introduction

Due to the increased public awareness about the water pollution problem, effluent standards for wastewater treatment plants (WWTPs) have become very strict [4,12]. These standards are also expected to be even stricter in the future [10]. The secondary settling tank (SST) plays a very important role in achieving such strict effluent quality standards. It is obvious that if the SST is not removing the suspended solids adequately; the standards will not be met.

The double-exponential function model [11] is a widely acceptable and used model for predicting the dynamic behavior of the SST. The ability of a model to describe adequately the dynamic behavior of a process, depend on (i) the model structure and (ii) the model parameter values. On the assumption that the model developed by Takács *et al*., (1991) [11] adequately describes the SST fundamental processes, an insight into the relative importance of the model parameters can be gained through sensitivity analysis.

The objective of this paper is to carry out sensitivity analysis in order to access the effect of parameter variations on the performance of the double-exponential function model for SST. Different sensitivity analysis methods are available [8]: (i) analytical or numerically approximated sensitivity, (ii) Taylor series expansion of the criterion function related to a parameter estimation problem and (iii) numerical simulations functions (e.g. the factorial sensitivity analysis method or Monte Carlo method). Among these methods, factorial sensitivity analysis method was used. The

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advantages of this method are the followings: (i) it is straightforward and (ii) it gives information about the interaction effect of the parameters [7,9,6].

2. Model description

Briefly, the double-exponential function model [11] is based on the principles of solid flux. It considers the SST to be a non-reactive, one-dimensional settler that consists of ten equal layers, with the feed inlet connected to the middle point of the 6th layer from the bottom. Flow flux in any layer is defined according to the layer location with reference to the feed inlet. Direction of the flow flux in the layers above the feed layer is considered upward, whereas that in the layers below the inlet is assumed downward. However, solid flux due to gravity sedimentation (downward flux) is assumed to take place in all the layers, including the bottom layer.

The main significant contribution of Takács *et al.* [11] is the correction of the settling model (Equation 1) used in the previous models, by adding a correction factor (second term in the right hand side of Equation 2) that accounts for the settling of smaller particles.

$$v_S = v_0 \cdot e^{-\alpha X} \tag{1}$$

$$v_{Sj} = v_0 \cdot e^{-r_h X_j^*} - v_0 \cdot e^{-r_p X_j^*} \tag{2}$$

Here X_j^* is defined as $\left(X_j - X_{\min}\right)$ with $X_{\min}$ equals $f_{ns} \cdot X_{in}$ and $0 \le v_{Sj} \le v_0'$. Further, v_S is the settling velocity (m d^{-1}), $X_{\min}$ is the minimum attainable suspended solids concentration in the effluent (g m^{-3}) and X_{in} is the mixed liquor suspended solids concentration (g m^{-3}) entering the SST. The rest of the variables are defined in Table 1. For more detailed information about the model the reader is referred to Takács *et al.* (1991).

Table 1: Default values of the parameters of the double-exponential SST model (Copp, 2002)

Parameter description	Parameter symbol	Default value
Maximum practical settling velocity (m d^{-1})	v_0'	250
Maximum theoretical settling velocity (m d^{-1})	v_0	474
Hindered zone settling parameter (m^3 g^{-1})	r_h	5.76e-4
Flocculant zone settling parameter (m^3 g^{-1})	r_p	2.86e-3
Non-settleable fraction (dimensionless)	f_{ns}	2.28e-3

3. Method

Sensitivity analysis was carried out for two models of different SSTs. The first model was for a

SST of a typical European WWTP, while the second model was for a SST of a large full-scale WWTP situated in Rotterdam, The Netherlands. According to Copp (2002) [2], a typical European WWTP consists of a pre-denitrification activated sludge reactor (anoxic volume=2000 m^3, aerobic volume=3999 m^3) and a SST (surface area=1500 m^2, depth=2 m). Such a plant is assumed to be operated at (i) dry weather inflow of 18446 m^3/d, (ii) recirculated activated sludge (RAS) rate of 18446 m^3/d and (iii) wasted activated sludge (WAS) rate of 385 m^3/d. Suspended solids load into the SST is estimated to be about 2.5 g m^{-3}. More information about this typical European plant can also be found at the web site of COST Action 624 (http://www.ensic.u-nancy.fr/COSTWWTP).

The full-scale SST studied here is for a large carrousel type WWTP that has two parallel treatment lines. Each treatment line consists of two primary settlers, one selector, one carrousel (13000 m^3) and three circular SSTs (surface area=2197.9 m^2, depth =2 m). At the study period, the plant was working at an inflow rate of 80254 m^3/d, RAS of 56178 m^3/d and WAS of 16051 m^3/d. Suspended solids load into the SST was about 3.1 g m^{-3} [3].

Two different SST simulation models, as suggested by Takács *et al.*, (1991) [11], were built in Matlab/Simulink as ten-layer non-reactive settlers. Note that in the case of the full-scale SST, the three SSTs in one treatment line were modeled as one large SST of surface area equals 6593.6 m^3 and depth equals 2 m.

A simulation experiment was then designed to fit the following second-order regression type meta-model of the solids concentrations (X_j) with respect to the five parameters of the model:

$$\begin{aligned} X_j &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 \\ &+ \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{44} x_4^2 + \beta_{55} x_5^2 \\ &+ \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{14} x_1 x_4 + \beta_{15} x_1 x_5 \\ &+ \beta_{23} x_2 x_3 + \beta_{24} x_2 x_4 + \beta_{25} x_2 x_5 + \beta_{34} x_3 x_4 + \beta_{35} x_3 x_5 + \beta_{45} x_4 x_5 \end{aligned} \tag{3}$$

where the coefficients (β_i and β_{ik}) represent the sensitivities and j equals 1 or 10 (i.e. the bottom or the top layer).

As recommended by Box and Draper (1987) [1], a two-level factorial design, with cubic, star and center was used to design the simulation experiment (Table 2). Note that all the points, except the center point, are situated on a ball in the parameter space. This design results in 43 (i.e. $2^5 + 5 \cdot 2 + 1$) different combinations of the parameter values. In other words, the simulation experiment consists of 43 different simulation runs.

Table 2: Coded level of the five parameters.

Parameter	Coded level, x_i					x_i in term of the parameters
	$-\sqrt{5}$	-1	0	+1	$+\sqrt{5}$	
$v_0^{'}$	170	225.3	270	314.7	370	$x_1=(v_0^{'}-270)/44.7$
v_0	150	246.7	325	403.3	500	$x_2=(v_0-325)/78.3$
r_h	2e-4	3.1e-4	4.0e-4	4.9e-4	6.e-4	$x_3=(r_h-4.0e\text{-}4)/8.9e\text{-}5$
r_p	2e-3	3.1e-3	4.0e-3	4.9e-3	6e-3	$x_4=(r_p-4.0e\text{-}4)/8.9e\text{-}5$
f_{ns}	1e-4	8.3e-4	15e-4	22e-4	30e-4	$x_5=(f_{ns}-15e\text{-}4)/6.7e\text{-}4$

* parameter ranges were obtained from the literature [2,4]

At the simulation stage, first, a 100-day steady-state simulation, using average values for the operating conditions, was performed in order to determine the initial solids concentrations in the various layers. Then, the 43 simulation runs described above were carried out for both models. In the case of the typical settler model, a 14-day data of the dry weather flow conditions were used. However, in the case of the full-scale settler model, daily measurements of performance of a real plant were used. For both sets of simulations, the average solids concentration in both the effluent (top layer) and the underflow (bottom layer), over the simulation period, were recorded and later analyzed using the meta-model described by Equation 6. This was done because the settling velocity, and consequently the gravity flux, depends upon the solids concentration (Equations 1 and 2). As there is a great difference between the solids concentration in the top layer and the bottom layer, one should therefore expect also great difference between the parameter sensitivities for these layers.

4. Results and discussion

Results of the sensitivity analysis of the typical SST are shown in Table 3. As can be seen from this table, suspended solids concentrations in the effluent stream can be approximated as:

$$X = \beta_0 + \beta_2 x_2 + \beta_4 x_4 + \beta_5 x_5 \tag{4}$$

This indicates that the parameters that significantly influence the prediction of the solids concentration in effluent stream are v_0, r_p and f_{ns}.

On the other hand, suspended solids concentrations in the underflow stream can be approximated as:

$$X = \beta_0 \tag{5}$$

However, this indicates that all the parameters do not significantly influence the prediction of the underflow concentrations. In turn, this clearly indicates that the model has a structural problem regarding the prediction of the solids concentrations in the underflow stream. It is known that sludge compaction at the bottom layers has not yet been included in SST models [5]. Inclusion of the compaction process in the model might remove the structural problem indicated here.

Table 3: The meta-models obtained for the typical SST

Coefficient	β value of the meta-model for the effluent concentration			β value of the meta-model for the under-flow concentration		
	Mean value ($g\ m^{-3}l$)	Standard deviation ($g\ m^{-3}l$)	C.V. (%)	Mean value ($g\ m^{-3}l$)	Standard deviation ($g\ m^{-3}l$)	C.V. (%)
β_0	**10.5**	**0.4**	**3.6**	**6408.5**	**1.9**	**0.03**
β_1	-0.01	0.1	350.9	0.9	0.3	32.86
β_2	**-2.2**	**0.1**	**2.7**	2.5	0.3	11.8
β_3	0.3	0.1	22.7	-0.1	0.3	294.8
β_4	**-2.0**	**0.1**	**2.8**	1.4	0.3	21.1
β_5	**1.3**	**0.1**	**4.3**	-1.6	0.3	18.4
β_{11}	-0.02	0.1	366.4	-1.5	0.5	30.4
β_{22}	0.6	0.1	14.0	-2.4	0.5	19.0
β_{33}	-0.04	0.1	243.6	-1.4	0.5	32.5
β_{44}	0.6	0.1	15.9	-1.7	0.5	26.8
β_{55}	0.03	0.1	325.1	-1.2	0.5	38.0
β_{12}	-0.01	0.1	762.1	0.1	0.3	337.9
β_{13}	0.004	0.1	1657.5	-0.2	0.3	169.0
β_{14}	-0.001	0.1	736.7	-0.3	0.3	112.6
β_{15}	0.003	0.1	2367.9	-0.2	0.3	169.0
β_{23}	-0.1	0.1	82.7	0.5	0.3	67.6
β_{24}	0.5	0.1	13.0	-0.6	0.3	56.3
β_{25}	0.1	0.1	101.2	-1.6	0.3	21.1
β_{34}	-0.1	0.1	55.1	0.03	0.3	985.1
β_{35}	-0.01	0.1	1163.2	0.5	0.3	67.6
β_{45}	0.1	0.1	111.6	0.3	0.3	112.6

* Significant numbers $(C.V. \leq 5\%)$ are printed in bold.

Figure 1 compares the variability of the suspended solids concentration in the effluent stream to that in the underflow stream. This figure also illustrates clearly the insignificant change in the underflow concentration due to the change in the parameter values.

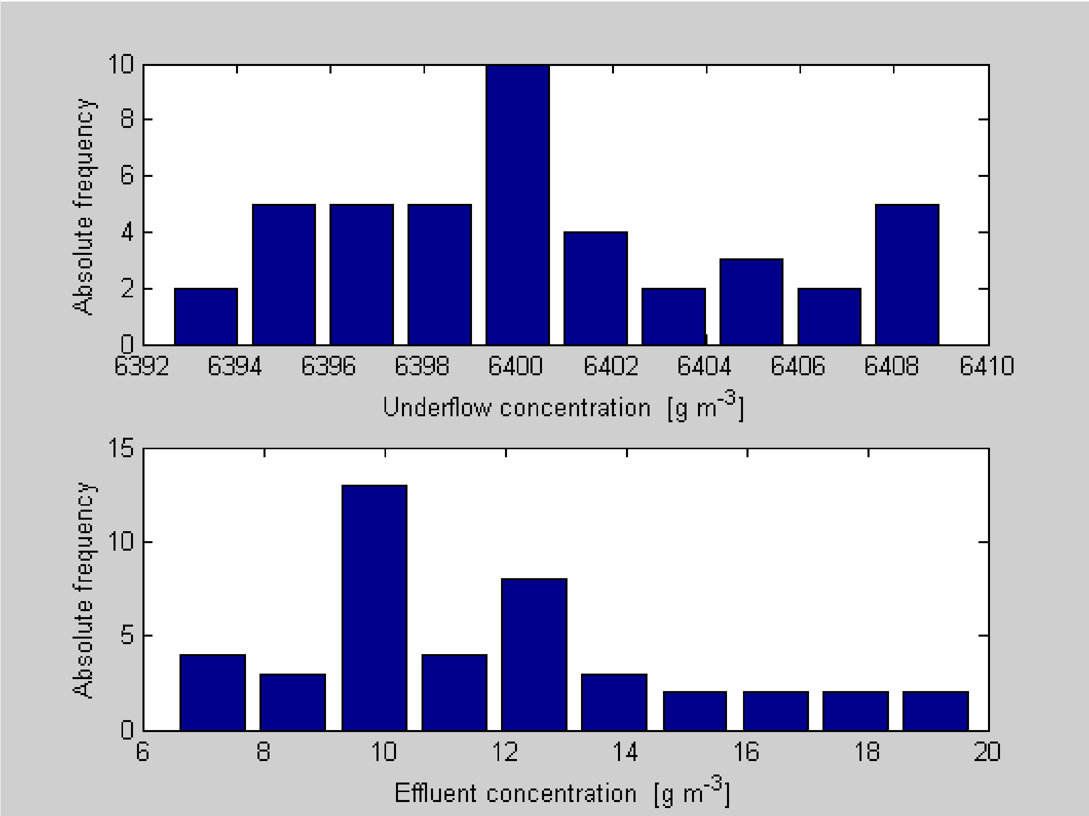

Figure 1: Comparison of the effluent concentration histogram with underflow concentration histogram.

Results obtained for the full-scale SST were similar to that of the typical SST (see Appendix). As before, suspended solids concentrations in the effluent stream can also be approximated by Equation (4), whereas that in the underflow stream can be approximated by Equation (5), which indicates that the model has an structural problem with respect to the prediction of the underflow concentrations.

Figure 2 illustrates the limitation of the double-exponential function model in predicting the suspended solids concentration in the underflow stream of a real SST. Figure 2 was obtained using *default* parameter values. As explained above, parameter values have insignificant effect on the prediction of the underflow concentration. Therefore, it is impossible to have a good prediction of such real measurements, irrespective of the parameter value used. This means that there is a need to modify or correct the SST model, in order to predict fairly the suspended solids concentration in the underflow stream.

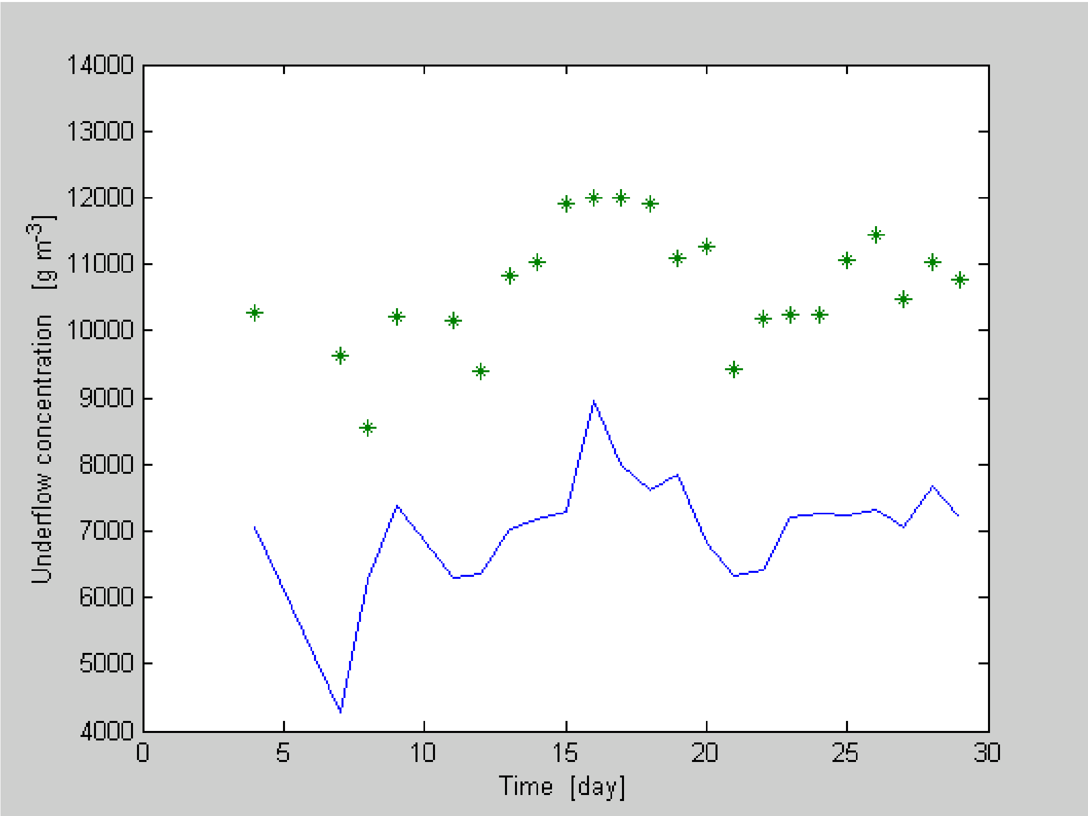

Figure 2: Comparison of the real measurements of underflow concentrations with the model predictions.

5. Conclusions

Factorial sensitivity analysis was carried out for a typical and a full-scale secondary settling tanks models in order to access the effect of the variations in the parameters of the double-exponential function secondary settling tank model [11] on the model behavior. Results obtained have indicated the following:

- Only the parameters v_0, r_p and f_{ns} significantly affect the prediction of the suspended solids concentration in the effluent stream.
- None of the parameters plays an important role in determining the suspended solids concentration in the underflow stream. This clearly indicates that the model has a structural problem regarding the prediction of the suspended solids concentration in the underflow stream.
- Parameter interactions play insignificant role in the model predictions.

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6. References

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7. Appendix

Table 4: The meta-models obtained for the full-scale SST

Coefficient	β value of the meta-model for the effluent concentration			β value of the meta-model for the underflow concentration		
	Mean value (g m^{-3}l)	Standard deviation (g m^{-3}l)	C.V. (%)	Mean value (g m^{-3}l)	Standard deviation (g m^{-3}l)	C.V. (%)
β_0	**5.8**	**0.1**	**2.1**	**7056.1**	**19.5**	**0.3**
β_1	-2.1e-6	0.02	210420	0.2	3.0	1507
β_2	**-0.6**	**0.02**	**3.0**	0.5	3.0	603
β_3	0.1	0.020	27.4	-0.5	3.0	603
β_4	**-0.6**	**0.02**	**3.0**	0.3	3.0	1005
β_5	**1.5**	**0.02**	**1.3**	7.9	3.0	38
β_{11}	-0.01	0.03	252.6	-2.7	4.7	172
β_{22}	0.2	0.03	14.9	-2.7	4.7	172
β_{33}	-0.01	0.03	293	-2.9	4.7	161
β_{44}	0.2	0.03	14.6	-2.6	4.7	179
β_{55}	0.1	0.03	33.2	15.5	4.7	30
β_{12}	2.4e-5	0.02	8.9e+4	0.2	3.5	1728
β_{13}	7.4e-6	0.02	2.9e+5	0.3	3.5	1152
β_{14}	-6.3e-6	0.02	3.5e+5	-0.3	3.5	1152
β_{15}	-1.3e-5	0.02	1.6e+5	0.1	3.5	3455
β_{23}	-0.02	0.02	95.2	0.01	3.5	34553
β_{24}	0.2	0.02	12.0	0.04	3.5	8638
β_{25}	0.1	0.02	25.1	0.03	3.5	11518
β_{34}	-0.04	0.02	57.9	-0.2	3.5	1728
β_{35}	-0.01	0.02	226.0	-0.7	3.5	494
β_{45}	0.1	0.02	25.1	-0.2	3.5	1728

* Significant numbers $(C.V. \leq 5\%)$ are printed in bold.